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# Developments in Probability-Based Strain-Life Analysis

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## ABSTRACT

An initial statistical approach to defining a probabilistic strain-life curve has been developed for material fatigue strength characterization in the low-cycle region. It is based on the statistics of the standard linear regression model, and assumes that the scatter of the base 10 log of low-cycle fatigue life is normally distributed, with a constant variance. Preliminary low-cycle fatigue data from a large-scale, experimental strain-life material test program were used to fit the probabilistic strain-life curve, using the probabilistic linear regression model derived herein. Upper and lower confidence bounds on the fatigue life prediction intervals were generated, with all of the test data falling within the 95% confidence region. The probabilistic strain-life model has the capability to be integrated into a Monte Carlo simulation of structural component fatigue life reliability analyses. The influence of two different ASTM low-cycle fatigue test specimen geometries on fatigue life scatter was also investigated. Crack growth in low-cycle fatigue tests, and the grain orientation of hourglass test specimens have both been shown to significantly affect the resulting scatter of fatigue life data at a given stress amplitude. The probabilistic linear regression model proves sufficient for characterizing the strain-life curve in the low-cycle fatigue region. More complex statistical methods that can account for non-constant fatigue life variance, and fatigue life runout data, must be utilized to probabilistically characterize the strain-life curve in the high-cycle fatigue region.

## INTRODUCTION

Shipboard operations of naval aircraft demand that structural integrity management ensure safety and readiness through a safe life concept. The underpinning of the Naval Air Systems Command's (NAVAIR) safe life methodology is the strain-life approach to predicting fatigue crack initiation under cyclic loading. Strain-life calculations are implemented in a service life-tracking program that computes Fatigue Life Expended (FLE) for critical structural components of naval aircraft. FLE is an index of structural life, such that at a value of 100% FLE, an air vehicle has a 1 in 1000 chance of having a 0.010 in. or larger crack in a structural component. Historically, an air platform was often removed from the inventory prior to achieving design service life. These early retirements of platforms were due to the fact that in the past, performance obsolescence was the leading cause of removal. Now, the naval aviation community may delay vehicle retirement to the point where the entire fleet of a particular aircraft type reaches 100% FLE. In other words, an individual aircraft will remain in the inventory until it reaches or slightly exceeds the 100% FLE limit, resulting in a fleet average retirement age of 100% FLE.

In order to manage air vehicles to these new, historically untested lifetime boundaries, the uncertainties must be clearly and quantitatively understood. The problem is that the current implementation of the strain-life approach in calculating FLE is primarily deterministic in nature.

The FLE calculation is based on a quasi-probabilistic argument and has only a limited account of the inherent variability of all of the design and operational parameters that contribute to fatigue failures in airframe components. To meet the challenge of evaluating life uncertainty in aging aircraft, a research effort is underway to extend the FLE approach with the development of a probability-based strain life model. The probabilistic model will quantify the variability in component fatigue lives, and will enable management to assess the overall structural reliability of an air vehicle. The first step in this process is the formulation of a probability-based strain life relationship.

### STRAIN-LIFE TEST DATA

The heart of the strain-life approach to predicting fatigue life is the Coffin-Manson equation (Eq. 1), which divides the total cyclic half-amplitude strain into elastic and plastic components, and derives fatigue life coefficient values from a linear regression of log transformed low-cycle fatigue (LCF) test data (Fig. 1).<sup>1</sup>

$$\Delta\epsilon/2 = \frac{\sigma'_f}{E}(2N_f)^b + \epsilon'_f(2N_f)^c \quad (1)$$

Fully reversed, strain-controlled low-cycle fatigue (LCF) and load controlled high-cycle fatigue (HCF) tests are performed to generate the data needed to derive a strain-life relation for a particular material. Standardized test procedures for conducting LCF tests are described in ASTM E606,<sup>2</sup> and in ASTM E466 for HCF tests.<sup>3</sup> Equation 1 represents a curve fit through the mean of the fatigue life data, but does not account for the scatter in fatigue life at a given strain amplitude. To do this, enough fatigue tests must be performed at various strain amplitudes to provide a reasonable estimate of fatigue life scatter, and to develop confidence bounds on the mean life estimates.

To quantify fatigue life scatter over a range of strain amplitudes, an experimental strain-life test program has been initiated, consisting of 410 hourglass and uniform gage section test specimens. All of the test specimens were machined from the same 0.50 in. thick 7050-T7451 aluminum plate. Thirty specimens are to be tested at each of thirteen different fully reversed, constant amplitude strain levels. Both low and high-cycle fatigue regimes are to be tested, with initial overstraining imposed on the high-cycle fatigue specimens to simulate plastic strain damage that would typically occur in a notched member under variable amplitude loading. A Randomized Complete Block (RCB) test matrix is specified, to ensure adequate randomization of test specimens and test sequences for the purpose of statistical data reduction. Preliminary LCF results from this test effort (Table 1) are used in the initial development of the probabilistic strain-life relation.

Strain controlled LCF specimens with low to moderate amounts of plastic strain exhibit crack growth over a portion of their lives. If not accounted for adequately, crack growth can skew the mean fatigue life curve, and increase the apparent scatter in fatigue life data. The NAVAIR definition of the end of useful structural life, based on safe-life fleet management criteria, is the presence of a 0.010 in. or greater crack in a metallic component. Therefore, cycles of crack growth beyond 0.010 in. should be subtracted from the total life of each fatigue test specimen. Failure progression in a typical LCF specimen is shown in Fig. 2, where the peak tensile load gradually decreases from its cyclically stabilized value as the crack progresses through the specimen cross-section. By applying a failure criteria based on peak load drop from the stabilized value, crack propagation can be subtracted from the total fatigue life for each

specimen. Exactly what peak load drop percentage value correlates to a 0.010 in. flaw size is currently unknown, and is likely a function of the test strain amplitude and specimen geometry.

**Table 1. Preliminary LCF Test Data at Various Strain Amplitudes, 7050-T7451 Al\***

Specimen Type	Strain Amplitude	Sample Size	Normal of Log Lives		Lognormal (Base 10)		
			Mean	SD	Mean	SD	COV
Hourglass	0.040	10	2.016	0.0963	107	23.9	0.225
	0.030	9	2.165	0.0930	150	32.4	0.217
	0.020	10	2.534	0.0976	351	79.8	0.228
	0.015	10	2.789	0.113	637	169	0.266
	0.012	10	3.012	0.0771	1040	187	0.179
	0.010	15	3.183	0.0794	1550	285	0.184
	0.008	15	3.425	0.0740	2700	464	0.172
Uniform Gage	0.010	5	3.064	0.0515	1170	139	0.119
	0.008	5	3.336	0.0366	2180	183	0.0843

\* based on 2% peak load drop failure criteria

From the test data (Table 1), failure progression for all of the uniform gage and hourglass specimens was statistically analyzed at a number of peak load drop percentages (Table 2).

**Table 2. Crack-Initiation Life Statistics Based on Percentage Load Drop Failure Criteria**

Specimen Type	Strain Amplitude	Failure Criteria	Normal of Log Lives		Lognormal (Base 10)		
			Mean	SD	Mean	SD	COV
Hourglass	0.015	2% LD	2.789	0.113	637	169	0.266
	0.015	5% LD	2.821	0.123	689	199	0.288
	0.015	10% LD	2.830	0.126	705	209	0.297
	0.015	Rupture	2.842	0.130	727	223	0.307
	0.008	2% LD	3.425	0.0740	2700	464	0.172
	0.008	5% LD	3.460	0.0866	2930	592	0.201
	0.008	10% LD	3.476	0.0915	3060	651	0.213
	0.008	Rupture	3.501	0.103	3260	786	0.241
Uniform Gage	0.008	2% LD	3.336	0.0366	2180	183	0.0843
	0.008	5% LD	3.351	0.0430	2250	224	0.0992
	0.008	10% LD	3.358	0.0461	2290	244	0.107
	0.008	Rupture	3.376	0.0505	2390	279	0.117

These results demonstrate that subtracting crack propagation life out of total life significantly reduces the mean and scatter of LCF data. Beyond about 15% peak load drop life, the differences in mean and scatter compared to total life were less significant. Based on these findings, a 2% load drop failure criteria was applied to all of the fatigue test data (Table 1). This value will be assumed to correspond to a 0.010 in. crack size, until further tests can be devised to correlate load drop percentages to specific crack sizes.

## LCF SPECIMEN TEST CONSIDERATIONS

When performing axial LCF tests for basic material characterization, ASTM E-606 (Ref. 2) details two types of specimen geometries that may be used. The first is a specimen with a constant diameter gage section, referred to herein as the uniform gage specimen. The second is a specimen with a constant radius of curvature in the gage section, referred to as the hourglass specimen (Fig. 3). Because of the variation in diameter along the loading direction of the hourglass specimen, axial strain cannot be measured directly. Instead, a diametral strain gage is used to measure the amount of contraction at the minimum specimen diameter, as a function of axial load. Diametral strain ( $\epsilon_d$ ) is then converted to axial strain ( $\epsilon$ ) using Equation 2, based on the assumption of a plastic Poisson's ratio of 0.5 (incompressible). This calculated value of axial strain is then used in the feedback loop to control the LCF test. Hourglass specimen tests shown here use a nominal value for elastic Poisson's ratio ( $\nu_e$ ) taken from MIL-HDBK-5 for 7050-T7451 plate.<sup>4</sup>

$$\epsilon = (\sigma / E)(1 - 2\nu_e) - 2\epsilon_d \quad (2)$$

The uniform gage specimen is recommended in ASTM E-606 (Ref. 2) for most LCF and HCF applications. The hourglass specimens are recommended for use in situations where the uniform gage specimens will buckle under compressive strain. Use of the hourglass specimen allows higher peak strain amplitudes to be achieved without buckling, enabling the testing of a greater range of strain amplitudes to characterize material fatigue behavior. To investigate the variation in fatigue lives resulting from the use of test specimens with differing geometries, a uniformity trial was conducted as part of the overall strain-life test program. Fifteen hourglass and five uniform gage specimens were each tested at strain half-amplitudes of 0.008 and 0.010 in/in. The dimensions of each type of specimen used in the tests are shown in Fig. 3. Test results are included as part of the preliminary LCF results (Table 1). These results show that the hourglass specimens have higher mean fatigue lives than the uniform gage specimens, but significantly more scatter for a given strain amplitude. Further investigation of the hourglass test data shows a significant correlation between the cyclically stabilized peak load value, and the resulting fatigue life for each specimen (Fig. 4). As the peak load value decreases, fatigue life increases noticeably. This trend was evident for all of the hourglass specimens tested, and becomes more pronounced as the test strain amplitude is increased. Two specimens previously tested at a 0.030 (in/in) strain half-amplitude were then selected for additional analysis, one giving the minimum fatigue life (H-865) and the other the maximum fatigue life (H-840). Both specimens were cross-sectioned in the grip area, and the exposed ends polished to examine the grain orientation through the cross-section (Fig. 5).

**Table 3. Grain Orientation Effect on Fatigue Life for Hourglass Specimens**

Specimen Number	Strain Amplitude	Grain Orientation	Peak Stress (Ksi)	Elastic Strain (in/in)	Plastic Strain (in/in)	2% Load Drop Life
H-865	0.030	LT	82.87	0.00820	0.0218	112
H-429	0.030	LT	82.37	0.00816	0.0218	112
H-840	0.030	ST	76.94	0.00762	0.0224	192
H-603	0.030	ST	76.75	0.00760	0.0224	222

A preferred grain orientation exists in the specimens examined, corresponding to the long-transverse (LT) and short-transverse (ST) directions of the raw plate that the specimens were machined from. Before cross-sectioning of the test specimens, a vertical notch was

machined into the specimen grip section at the location of the cross-section, to allow the grain orientation to be keyed to the contact locations of the diametral strain gage. The grain orientation with respect to diametral strain gage placement is shown in Fig. 6 for the two specimens examined. The effect that grain orientation has on the hourglass fatigue lives is evident from the test data results (Table 3).

Also included in Table 3 are two specimens (H-429 & H-603) that were cross-sectioned prior to testing to determine grain orientation. These specimens were then tested with the orientations controlled as shown to validate the trends observed in the analysis of specimens H-840 and H-865. The results show that controlling grain orientation during hourglass testing can significantly reduce the observed fatigue life scatter when testing at large plastic strain amplitudes. Comparing the stabilized hysteresis loops of hourglass test specimens to those of uniform gage specimens tested at the same strain amplitude gives a good indication of the range of test error associated with the diametral-to-axial strain conversion equation (Eq. 2). This was done using the minimum and maximum fatigue life hourglass specimens at 0.010 in/in strain amplitude. The result was that the stabilized hysteresis loop for the long life hourglass specimen closely matched that of the uniform gage specimens, while the loop for the short life hourglass deviated significantly from the uniform gage loop shape. Accepting the prior result that the long life hourglass specimens correspond to a test performed in the S-T grain direction, it can be deduced that the axial strain calculation for the L-T grain direction tests is the source of the hourglass testing error. Since all of the hourglass tests except specimens H-429 & H603 were performed with the grain direction oriented randomly with respect to the strain gage placement, this testing error is included in the overall fatigue life scatter of the specimens. Therefore, the fatigue life scatter results derived from Table 1 data are a conservative estimate of the true fatigue life scatter of the base material.

### PROBABILISTIC STRAIN-LIFE MODEL

The preliminary LCF test results (Table 1) are sufficient to provide data for an initial evaluation of a probabilistic strain-life curve in the low-cycle region of life. The form of linear regression model used to curve fit the fatigue data is given in Equation 3, where the independent variable is the strain half-amplitude ( $\Delta\epsilon/2$ ), the dependent variable is the number of reversals to failure ( $2N_f$ ), and gamma is the model error, which is assumed to be normally distributed about the mean regression line, with a constant variance.

$$\log_{10}(2N_f) = A + B \log_{10}(\Delta\epsilon/2) + \gamma \quad \gamma \sim N(0, \sigma^2) \quad (3)$$

$$b = 1/B \quad \text{or} \quad c = 1/B \quad (4)$$

$$\sigma'_f / E = 10^{-A/B} \quad \text{or} \quad \epsilon'_f = 10^{-A/B} \quad (5)$$

A regression curve fit of the LCF dataset is shown in Fig. 7. A bimodal regression of the elastic and plastic strains is used, as this gives the best fit to the strain-life data. The mean fatigue life data points at each test strain amplitude are plotted to show the correlation between the test data and the regression curves. Stress-life data from Ref. 4 are also plotted to show the behavior of the strain-life curve in the HCF region. The elastic and plastic inflection points are determined by the points where the two regression curves intersect. Based on the inflection point location, the total strain-life curve can be divided into three regions, as shown in Fig. 7. Region I is the LCF region, where plastic strain dominates the fatigue behavior. Region II is a transition region between LCF and HCF, where plastic strains are still present, but at magnitudes smaller than the

elastic strains. Region III is the HCF region, where plastic strains are negligible, and the data are obtained using load-controlled tests. Also noteworthy is the change in fatigue life scatter when transitioning from Region I to Region II, which can be seen in the data of Table 2.

A notional procedure for developing a probabilistic strain-life relation within the framework of the Coffin-Manson equation can be derived from linear regression theory, and is greatly simplified by the assumption that the regression errors are normally distributed, with a constant variance over the regression range. Given the definitions for the log-log transformed variables and means in equation 6, the standard linear least-squares estimators for the regression model form of equation 3 are shown in equation 7, for a sample size of  $n$  data points.

$$x = \log_{10}(\Delta\varepsilon/2), \quad y = \log_{10}(2N_f), \quad \bar{x} = \sum_{i=1}^n x_i, \quad \bar{y} = \sum_{i=1}^n y_i \quad (6)$$

$$\hat{A} = \bar{y} - \hat{B}\bar{x} \quad \text{and} \quad \hat{B} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} \quad (7)$$

The unbiased estimator for the error variance term  $\sigma^2$  in equation 3 is the sample error variance of equation 8.

$$s^2 = \frac{\sum_{i=1}^n (y_i - \hat{A} - \hat{B}x_i)^2}{n-2} \quad (8)$$

If the model (Eq. 3) is rewritten in matrix form (Eq. 9), the variance-covariance matrix of the model parameter estimators  $\beta$  can be found through equation 10.<sup>5</sup>

$$Y = X\beta + \varepsilon \Rightarrow \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{Bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{Bmatrix} \hat{A} \\ \hat{B} \end{Bmatrix} + \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{Bmatrix} \quad (9)$$

$$\text{Var}(\beta) = \sigma^2 [X'X]^{-1} = \sigma^2 \begin{bmatrix} \frac{1}{n} + \frac{\bar{x}^2}{S_{xx}} & -\frac{\bar{x}}{S_{xx}} \\ -\frac{\bar{x}}{S_{xx}} & \frac{1}{S_{xx}} \end{bmatrix} \quad S_{xx} = \sum_{i=1}^n (x_i - \bar{x})^2 \quad (10)$$

Because of the normal error assumption made previously, the marginal distributions of the model parameter estimators are also normal, with the variances defined in equation 10.

$$\hat{A} \sim N[A, \sigma^2(1/n + \bar{x}^2/S_{xx})] \quad \hat{B} \sim N[B, \sigma^2/S_{xx}] \quad (11)$$

However, the model parameter estimators are also correlated through the covariance term of equation 10. It is advantageous to express one of the model parameter distributions as a conditional function of the other. This would allow random simulations to be performed on both estimators that would take into account the correlation between the two terms. The derivation can be accomplished by assuming the joint distribution of the model parameter estimators is bivariate normal. The sample error variance (Eq. 8) can also be substituted for the true error variance term  $\sigma^2$  in the normal distributions of equation 11 (Ref. 5). The resulting transformed random parameters are Student-t distributed with  $n-2$  degrees of freedom, and with the random estimator for  $A$  conditional on the random value of the estimator for  $B$  (Eqns. 12 & 13).

$$\frac{(\hat{B} - B)}{s/\sqrt{S_{xx}}} \sim t_{n-2} \quad (12)$$

$$\frac{\hat{A} - A + \bar{x}(\hat{B} - B)}{s/\sqrt{n}} \sim t_{n-2} \quad (13)$$

The above formulation has the advantage of accounting for the finite sample sizes of fatigue test data, where there is a strong incentive to minimize the required number of tests due to cost and schedule considerations. The random values of the model parameter estimates of  $A$  and  $B$  can be easily transformed to values for the fatigue life model parameters in the Coffin-Manson equation by applying equations 4 & 5, for both the elastic and plastic strain regressions. This gives a random distribution of *mean* fatigue life at a given strain amplitude, but the random variation of fatigue lives *about* the mean must still be taken into account. Going back to the assumption of normally distributed errors in the linear regression, the log of the fatigue lives will be distributed normally about the mean, with a variance equal to the true error variance term  $\sigma^2$ . Substituting in the sample error variance (Eq. 8) for the true error variance in a manner similar to Equations 12 & 13, the distribution of fatigue life about the randomized mean value, for a given strain amplitude value of  $x_j$ , is Student-t, with  $n - 2$  degrees of freedom.

$$\frac{y_j - \hat{A} - \hat{B}x_j}{s} \sim t_{n-2} \quad (14)$$

The Student-t distributions of Equations 12-14 are now all that is needed to characterize a probabilistic strain-life curve in the LCF region. For any value of strain half-amplitude in the LCF region, a random value of fatigue life can be generated for use in a Monte Carlo simulation of component fatigue reliability. Upper and lower confidence bounds can be assigned to the prediction intervals of the probabilistic strain-life model, to illustrate the range of variation in simulated fatigue lives due to scatter in the fatigue test data. A 95% confidence prediction interval is shown in Fig. 8, based on 5000 Monte Carlo simulations at each test strain amplitude. The effect of the bimodal regression is evident on the prediction intervals, in that the fatigue test data in Region II has less relative scatter compared to the data in Region I, hence the prediction intervals in Region II are more closely bounded.

The probabilistic strain-life methodology developed here is applicable when the fatigue test data indicate that the scatter is relatively constant over the range of regression, meaning that enough fatigue tests need to be performed at several strain amplitudes to verify that this is indeed the case. The number of fatigue tests necessary to quantify the range of scatter at each strain amplitude is an unknown variable. For this preliminary analysis, it was assumed that 10 tests at each strain amplitude are sufficient to characterize scatter. Additional data points from ongoing LCF tests will serve to validate whether this is an accurate assumption. In the HCF region, fatigue life variance is typically non-constant, and also includes censored data in the form of runout tests which were halted before failure. The regression methods presented here are not sufficient to characterize fatigue data under those conditions, and more complex methods, such as those of Nelson,<sup>6</sup> or Pascual and Meeker,<sup>7</sup> must be utilized for high-cycle fatigue data.

## CONCLUSIONS

A properly characterized, probabilistic strain-life curve of material fatigue strength is essential for reliability-based fatigue life predictions of aircraft components. Crack growth in



low-cycle fatigue tests, and the grain orientation of hourglass test specimens have both been shown to significantly affect the resulting scatter of fatigue life data at a given stress amplitude. The additional scatter associated with testing error of the hourglass specimens may be acceptable, as long as it is understood that the resulting fatigue life scatter values are conservative estimates of the true fatigue life scatter of the material. The sensitivity of a probabilistic structural component fatigue life analysis to material strain-life mean and scatter parameters is currently unknown. Therefore, further research should be done on the fatigue life scatter of hourglass test specimens compared to uniform gage test specimens, and on whether the additional expense and difficulty of testing hourglass specimens is justified in light of the resulting sensitivity to a component fatigue life analysis solution.

The notional approach that has been developed here to characterize a probabilistic strain-life relation is considered acceptable for use in the low-cycle region of fatigue life, where the scatter of the log of the fatigue lives is normally distributed, with constant variance. However, this may not be the best or most efficient method of characterizing the probabilistic strain-life curve. Further investigation needs to be done to identify statistical methods that can improve on or simplify the regression method shown here, and that can account for test cases that violate the statistical assumptions made in the probabilistic regression derivation.

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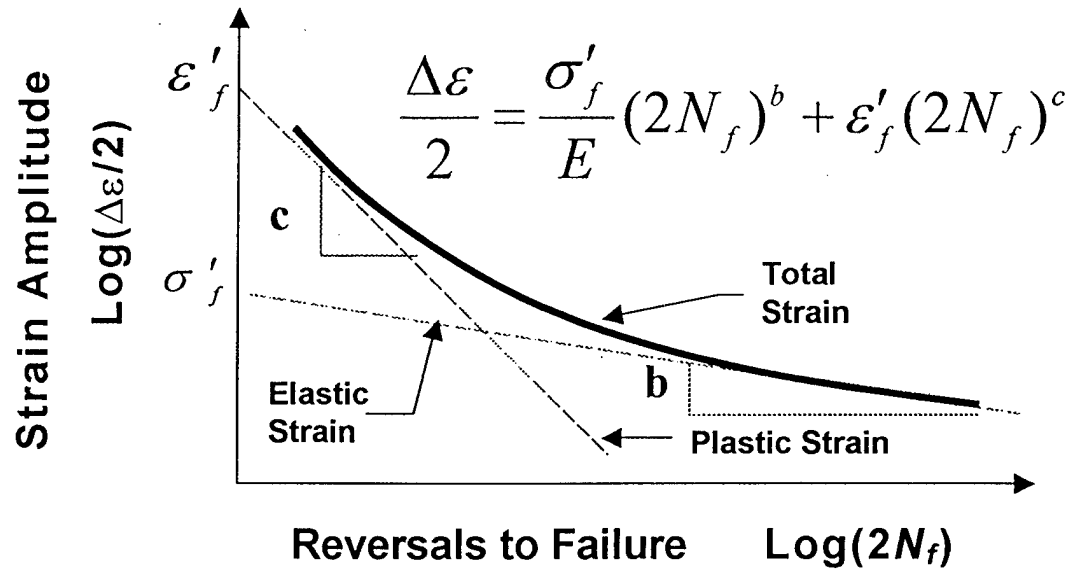


Figure 1. Strain-Life Relation for Material Fatigue Resistance

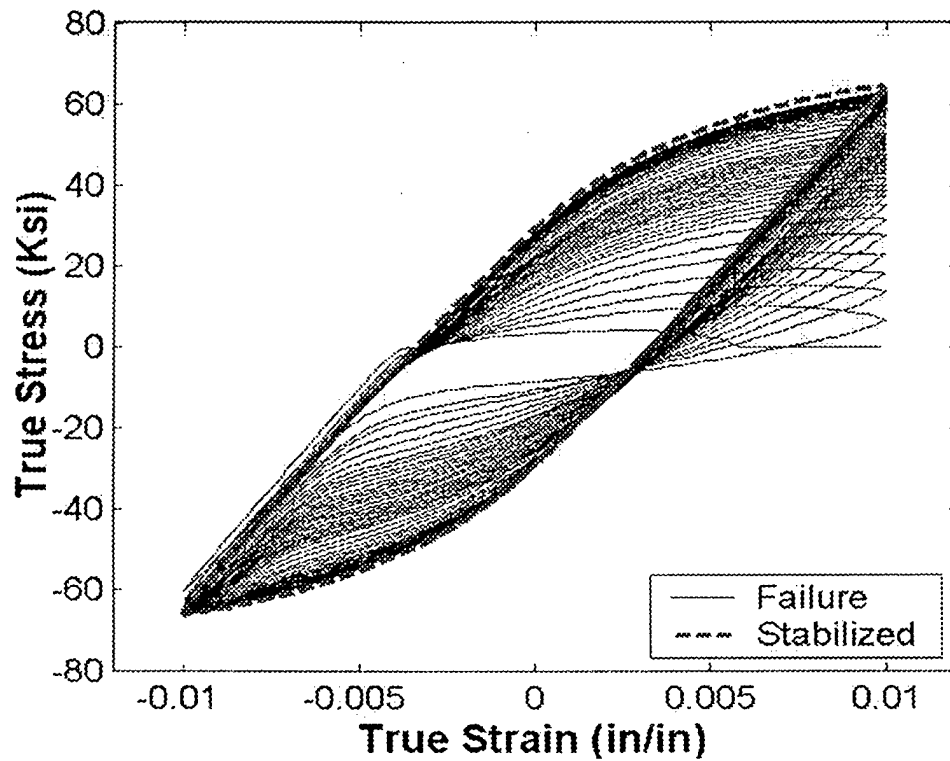


Figure 2. Hysteresis Loop Failure Progression in ASTM E-606 LCF Uniform Gage Specimen

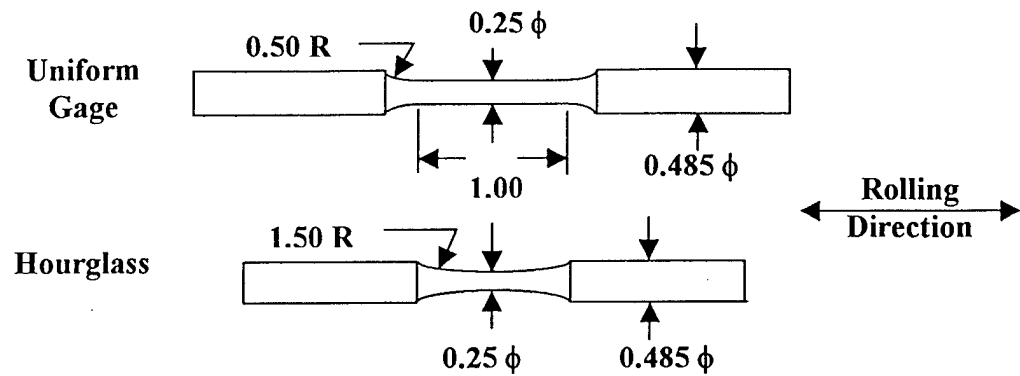


Figure 3. ASTM E-606 Uniform Gage and Hourglass Test Specimen Geometry

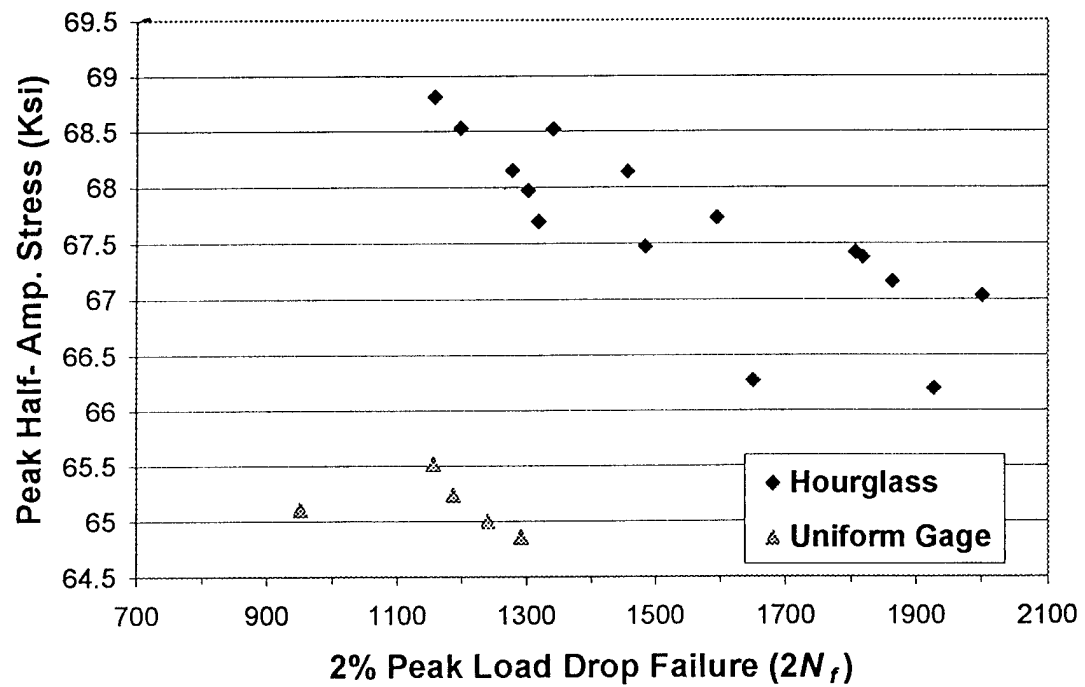


Figure 4. Peak Stress vs. Crack Initiation Life at 0.010 (in/in) Strain Half-Amplitude

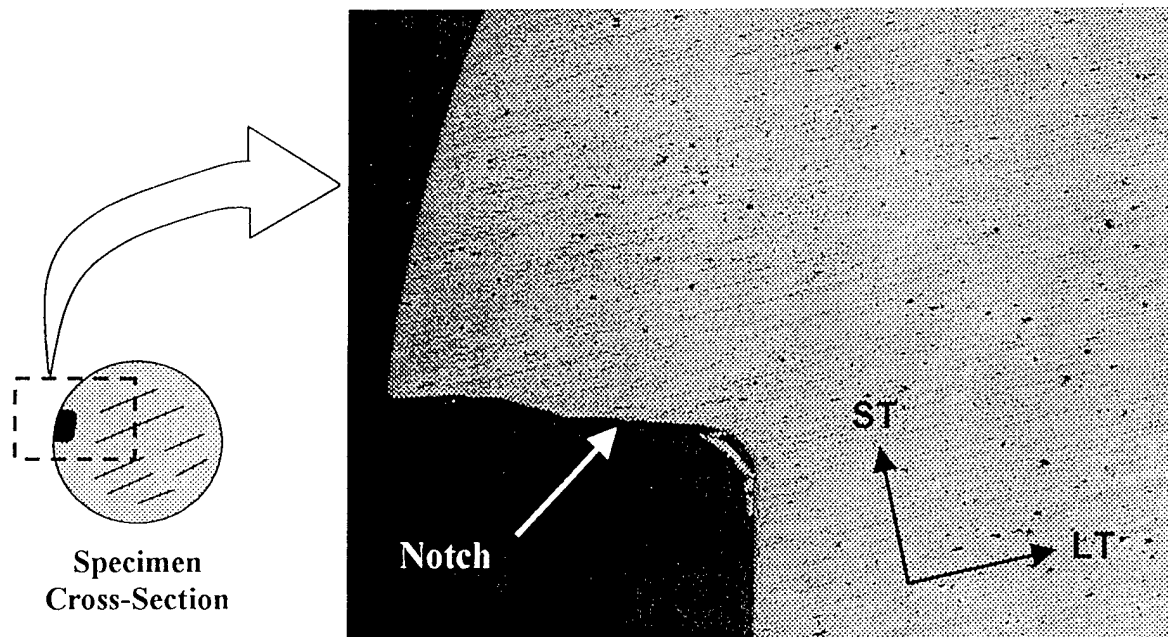


Figure 5. Grain Orientation of Cross-Sectioned 7050-T7451 Hourglass LCF Test Specimen

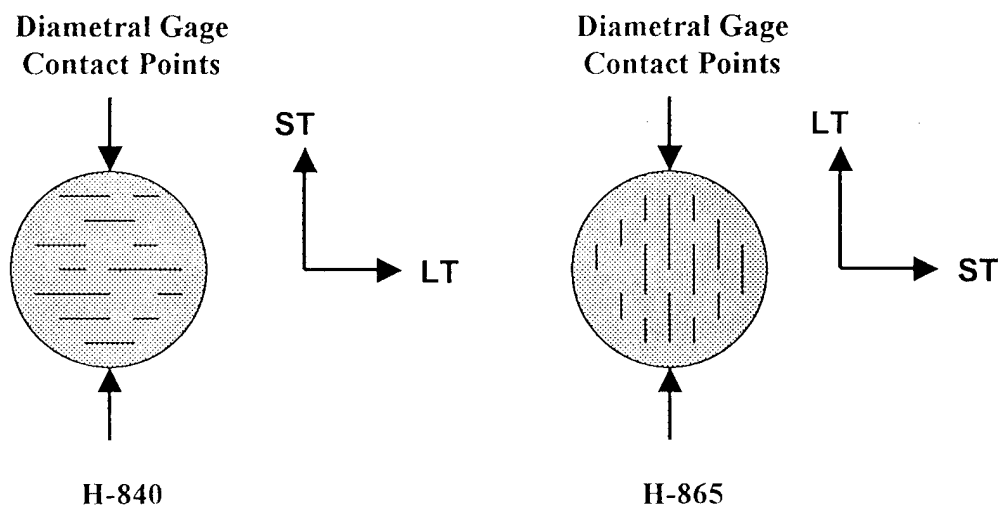


Figure 6. Grain Orientation With Respect To Strain Gage Placement for Two Hourglass Test Specimens

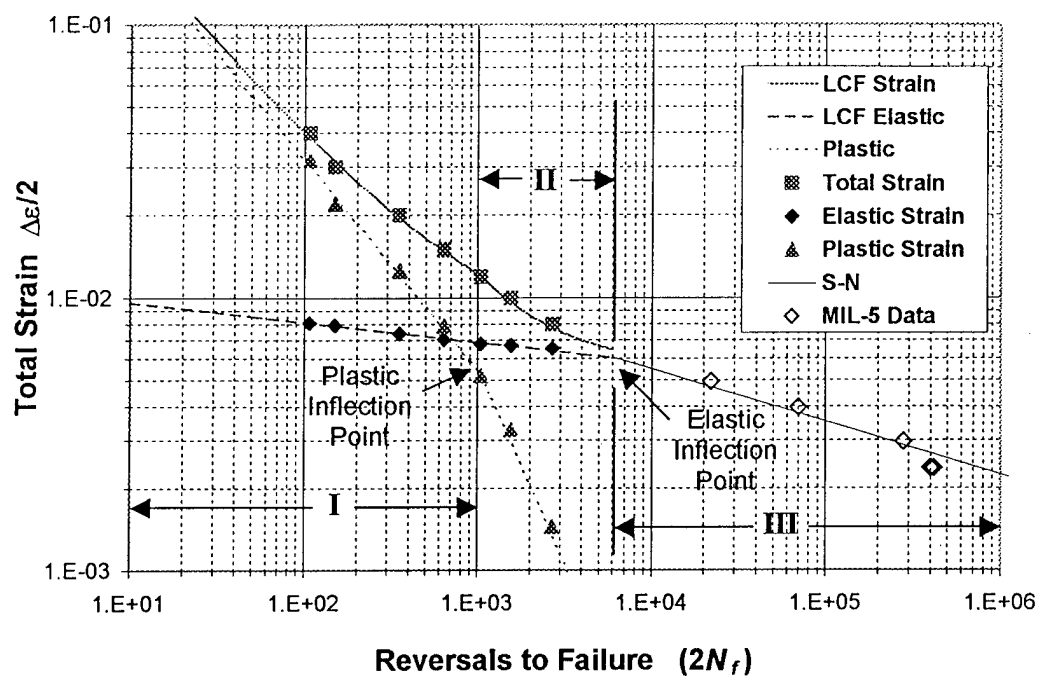


Figure 7. Bimodal Regression Curve Fit of Mean Hourglass Strain-Life Data

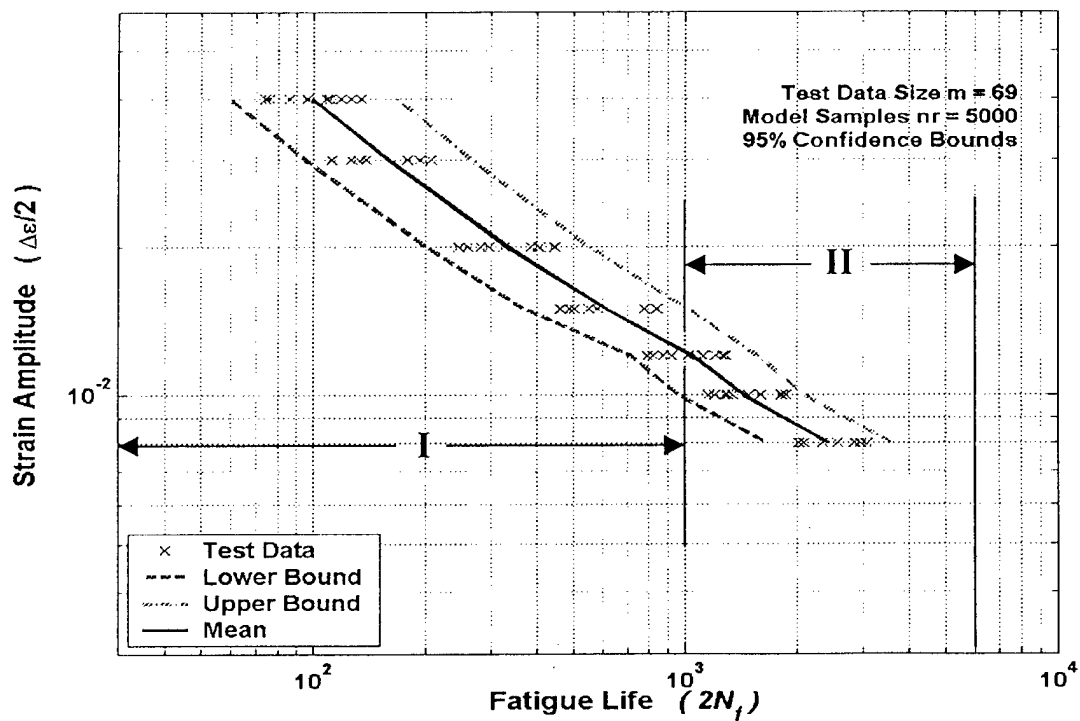
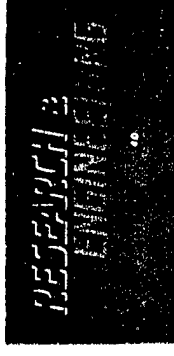


Figure 8. Prediction Intervals for Probabilistic Strain-Life Model in the LCF Region



**NAVAIR Structures Division**  
*AIR 4.3.3 Science & Technology Team*



# **DEVELOPMENTS IN PROBABILITY- BASED STRAIN-LIFE ANALYSIS**

**Presented To**

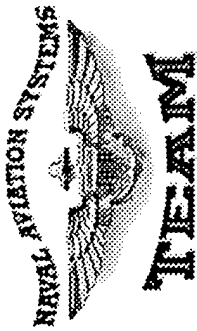
**Fifth Joint NASA/FAA/DoD Aging Aircraft  
Conference**

**11 September 2001**

**Principal Investigators**

**David Rusk, P.E.**

**Dr. Paul Hoffman, P.E.**



## Airframe Structural Life Management Criteria

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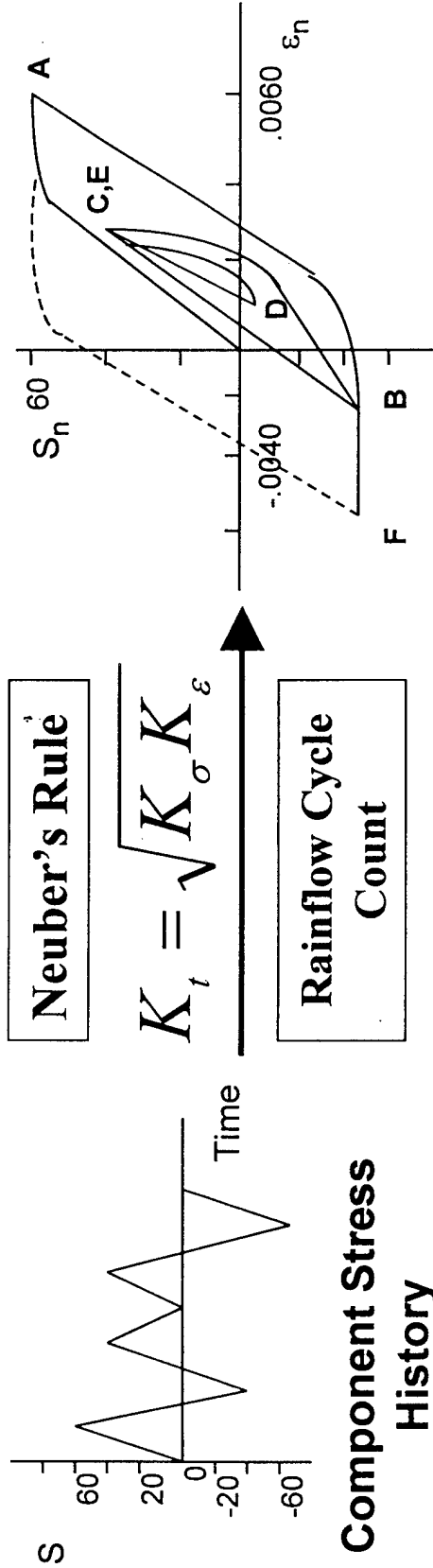


- **SAFE LIFE:** Aircraft are presumed to be “crack free” when new. Aircraft are retired/repaired before the onset of structural cracks. End of useful structural life is defined as 1/1000 probability of 0.010” or larger crack size. [NAVAIR]
- **FATIGUE LIFE EXPENDED (FLE):** Index of structural life. Historical aircraft retired at less than 100% FLE due to performance obsolescence. New requirements are to keep aircraft in inventory to 100% FLE or greater.
- **Note:** *Never before (Cold War Era) have we preemptively reached the operational life limits of the structure!*

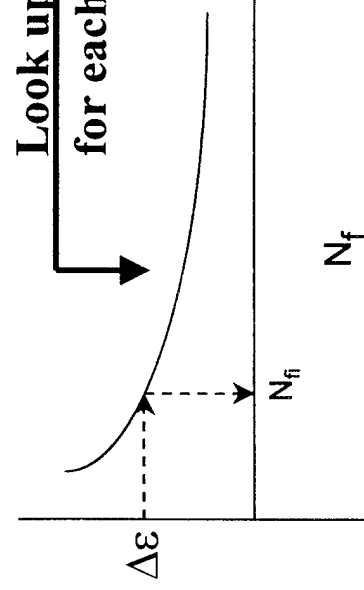




# Strain-Life Implementation



Look up Crack Initiation Life for each strain amplitude,  $\Delta\epsilon$



Hysteresis Loops

$$\text{Damage} = \sum \frac{n_i}{N_{fi}}$$

Compute

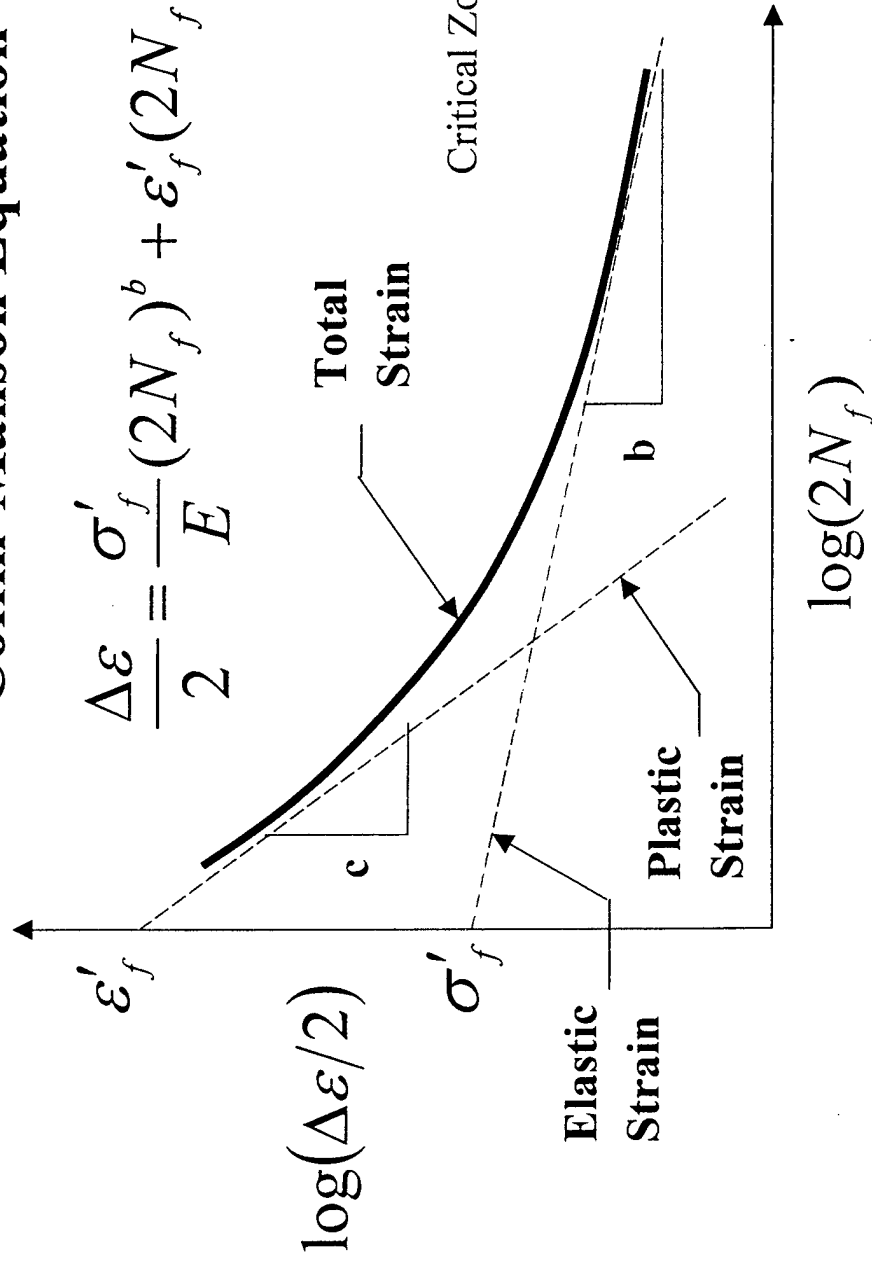
$$\text{FLE} = 2 * \text{Damage} * 100\%$$

Life Determination From Equivalent Strain Amplitudes

# Strain-Life Curve

## Coffin-Manson Equation

$$\frac{\Delta \varepsilon}{2} = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c$$





# Probabilistic Strain-Life Test Program

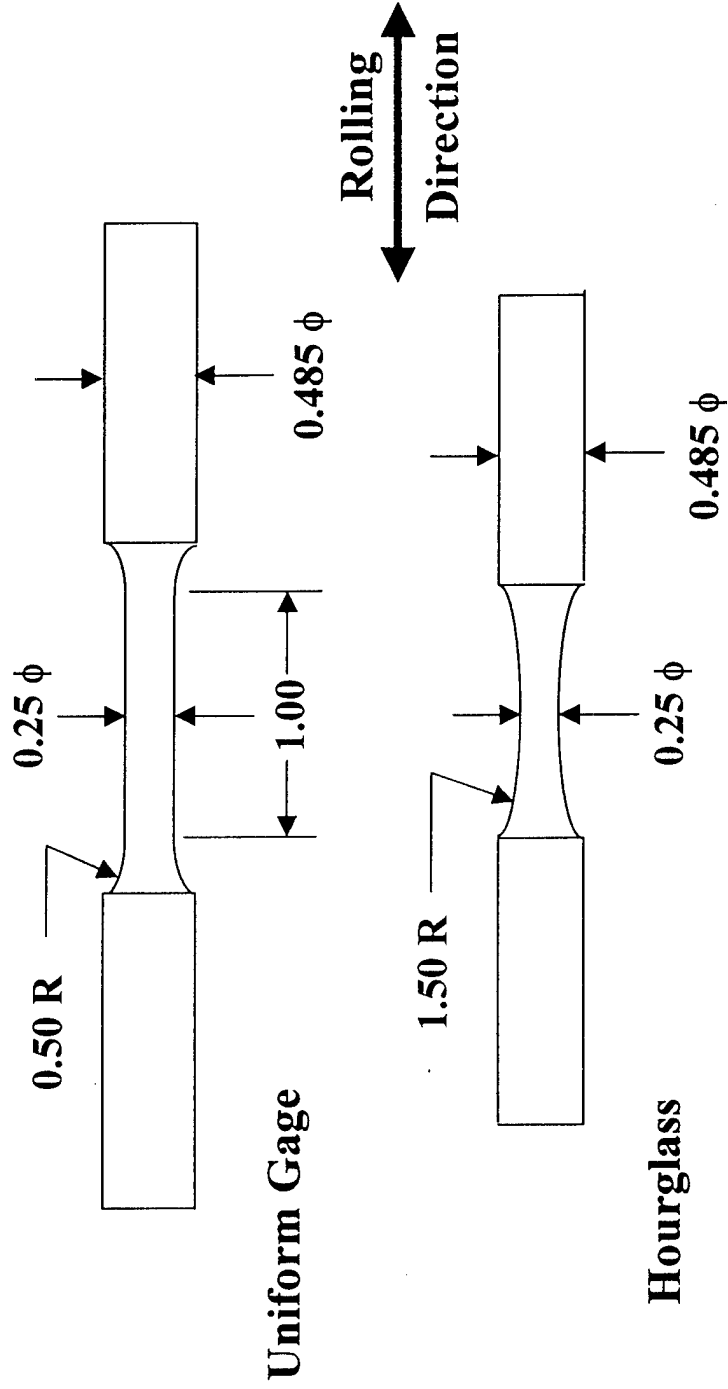


Goal: Quantify Scatter in Strain-Life Curve

- 13 Strain Amplitudes, 30 Tests Each, 410 Specimens
- 2 Test Labs (1 Government, 1 Contractor)
- Statistical Design of Experiment (DOE), per ASTM STP 588,  
*Manual on Statistical Planning and Analysis for Fatigue Experiments*
- Randomized Complete Block (RCB) Test Matrix, Blocked on:
  - Lab
  - Strain Level
  - Specimen Type (Uniform Gage & Hourglass)
- Uniformity Trial: LCF, 2 Strain Levels, 2 Specimen Types
- LCF Population: 7 Strain Levels, Hourglass Specimens
- HCF Population: 4 Strain Levels, Hourglass Specimens

# Probabilistic Strain-Life Test Program

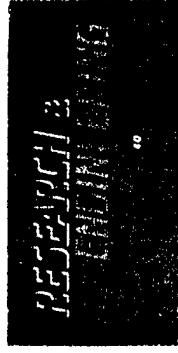
Material: 7050-T7451 Aircraft Aluminum, 0.50" thick plate



ASTM E606 LCF Test Specimens



# Fatigue Life Scatter

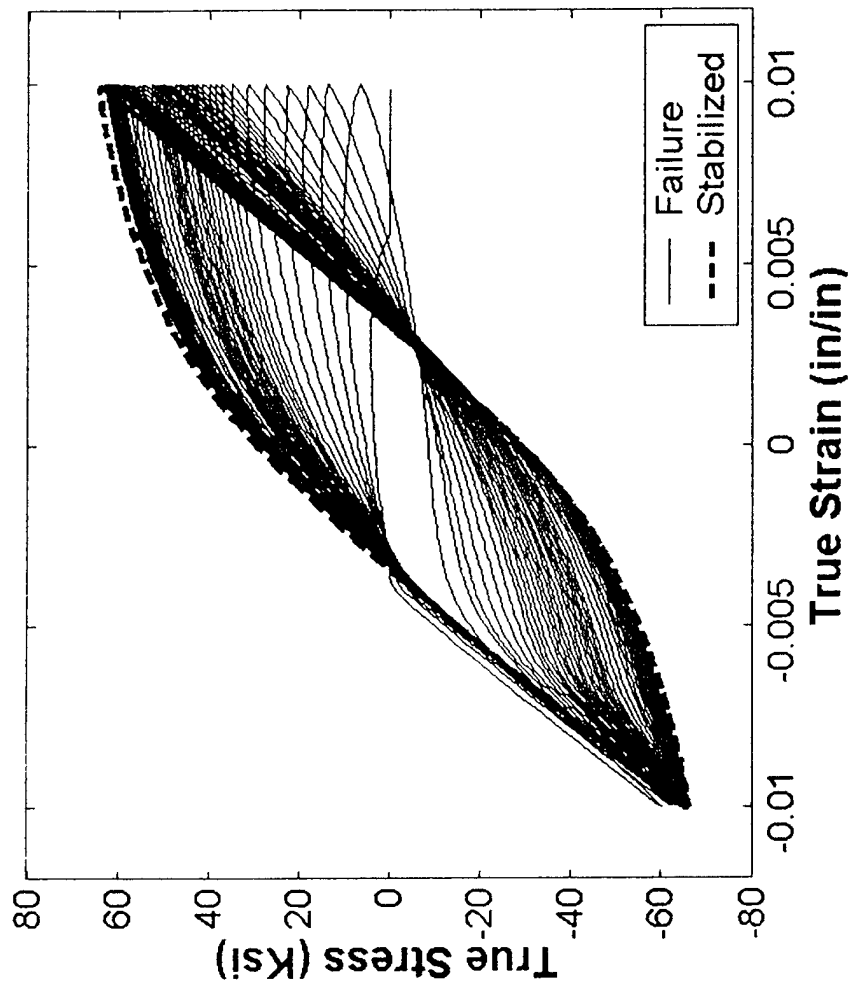


## Preliminary LCF Test Data, 2% Load Drop Failure

Specimen Type	Strain Amp.	Sample Size	Normal of Log Life		Lognormal (Base 10)		
			Mean	SD	Mean	SD	COV
Hourglass	0.040	10	2.016	0.0963	107	23.9	0.225
	0.030	9	2.165	0.0930	150	32.4	0.217
	0.020	10	2.534	0.0976	351	79.8	0.228
	0.015	10	2.789	0.113	637	169	0.266
	0.012	10	3.012	0.0771	1040	187	0.179
	0.010	15	3.183	0.0794	1550	285	0.184
	0.008	15	3.425	0.0740	2700	464	0.172
Uniform Gage	0.010	5	3.064	0.0515	1170	139	0.119
	0.008	5	3.336	0.0366	2180	183	0.0843

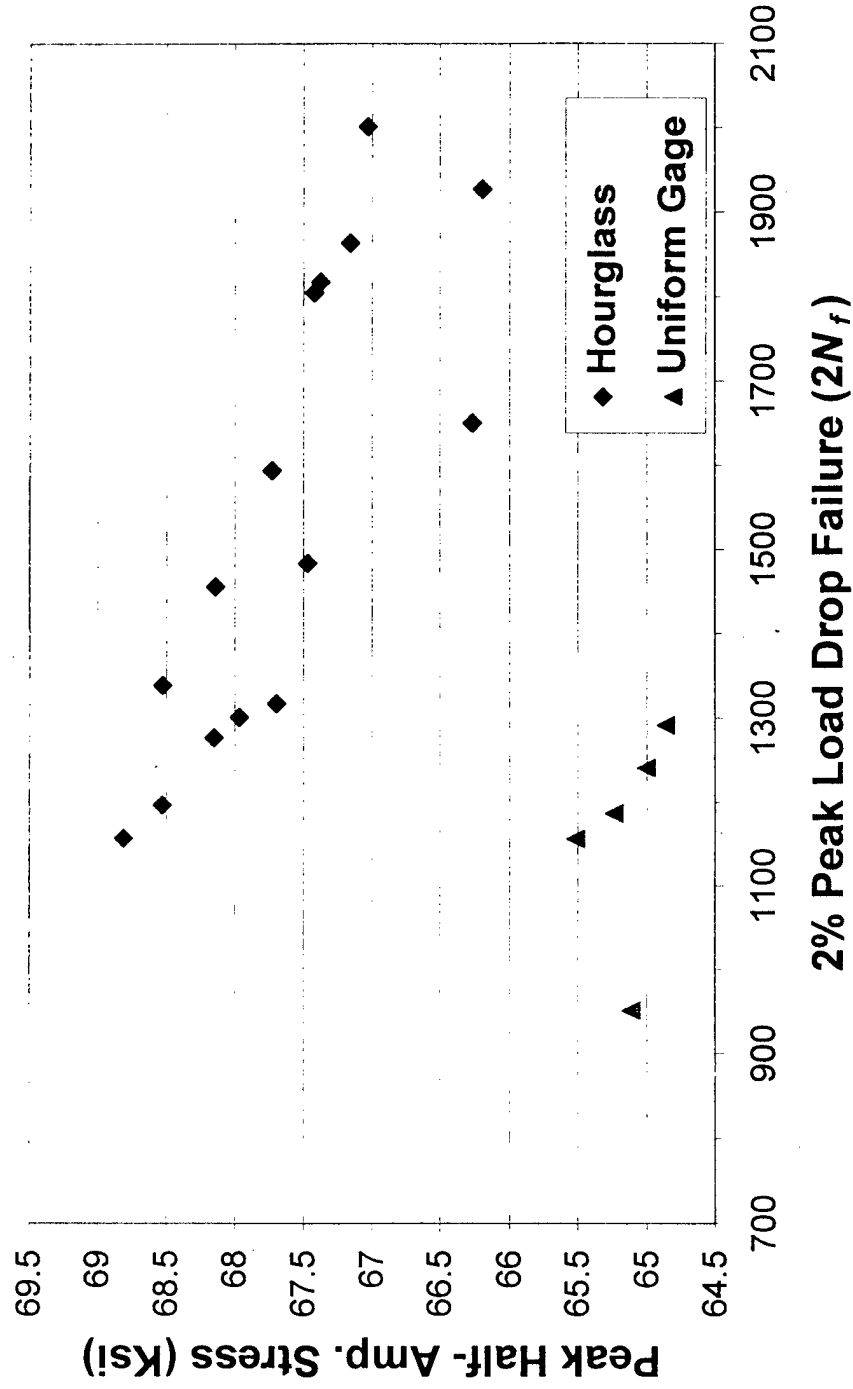
# Cyclic Failure Progression

## Hysteresis Loop Failure Progression for Uniform Gage Specimens

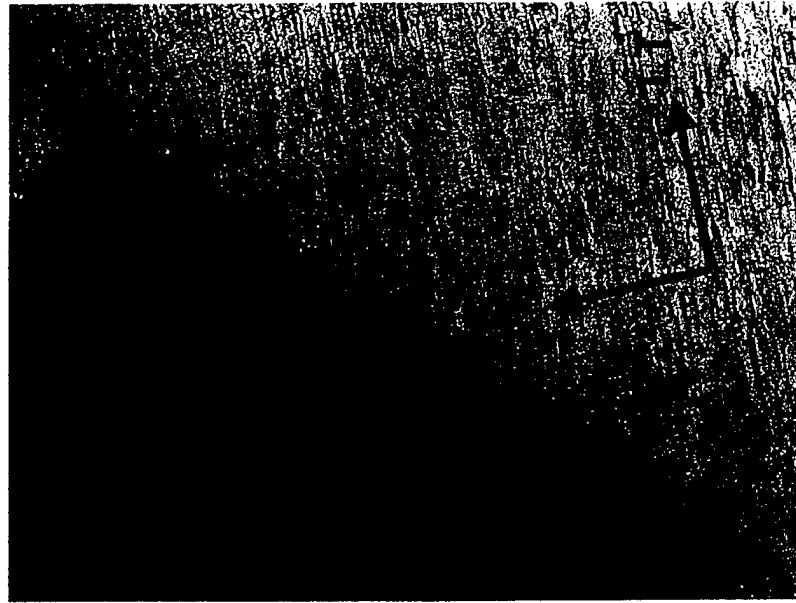


# Hourglass Fatigue Life Scatter

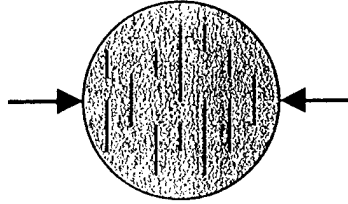
Peak Stress vs. Life at 0.010 (in/in) Strain Amp.



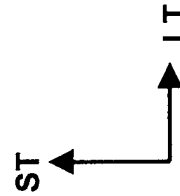
# Hourglass Specimen Grain Orientation



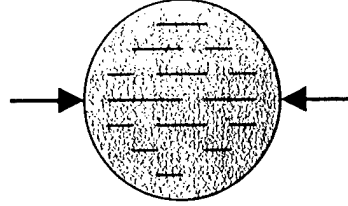
Diametral Gage  
Contact Points



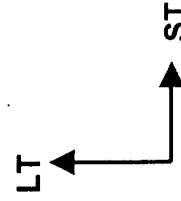
H-840



Diametral Gage  
Contact Points



H-865

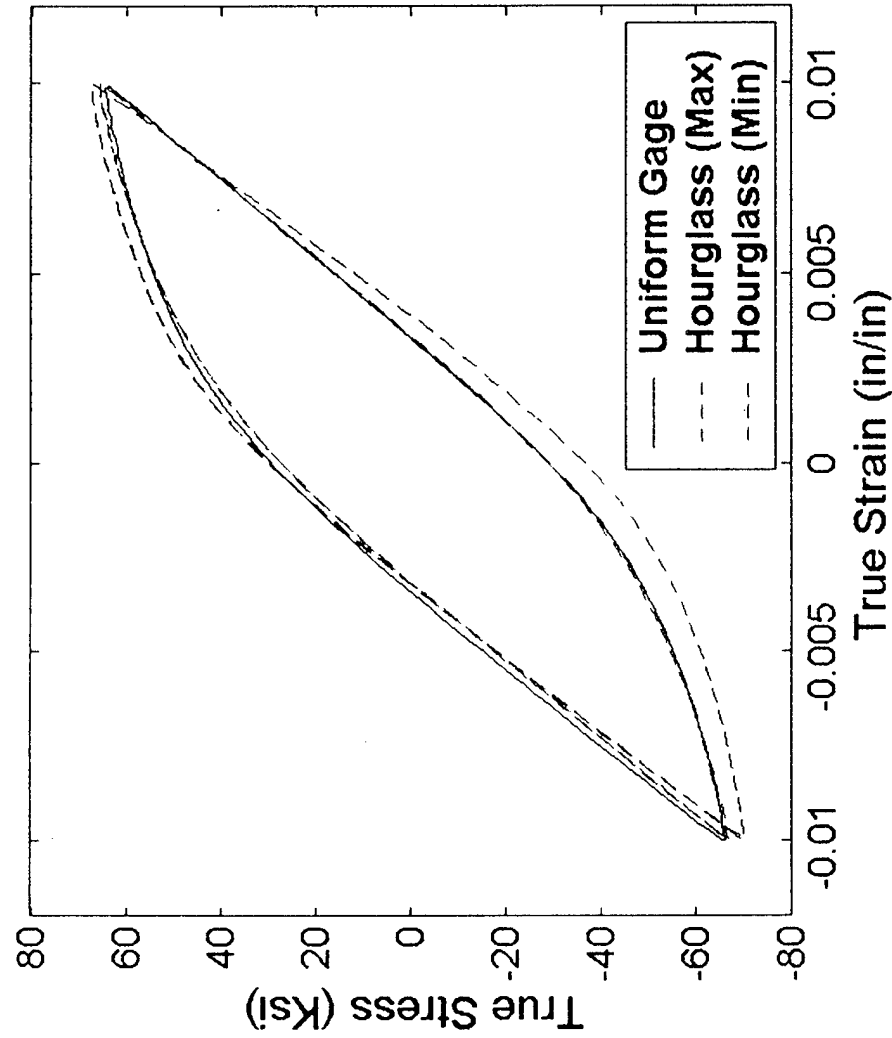




# Peak Load Variation

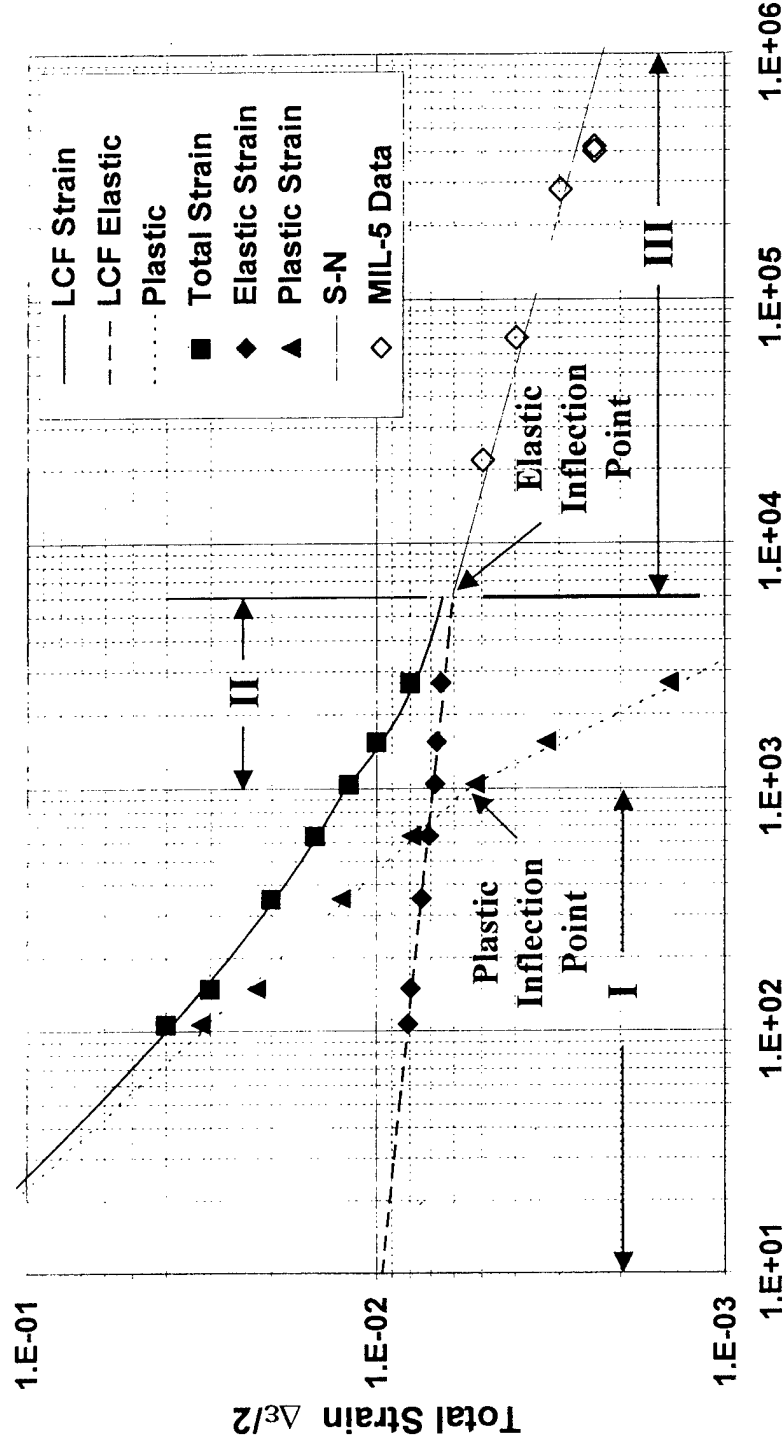


## Hysteresis Loop Variation for Hourglass Specimens



# Bimodal Strain-Life Curve Fit

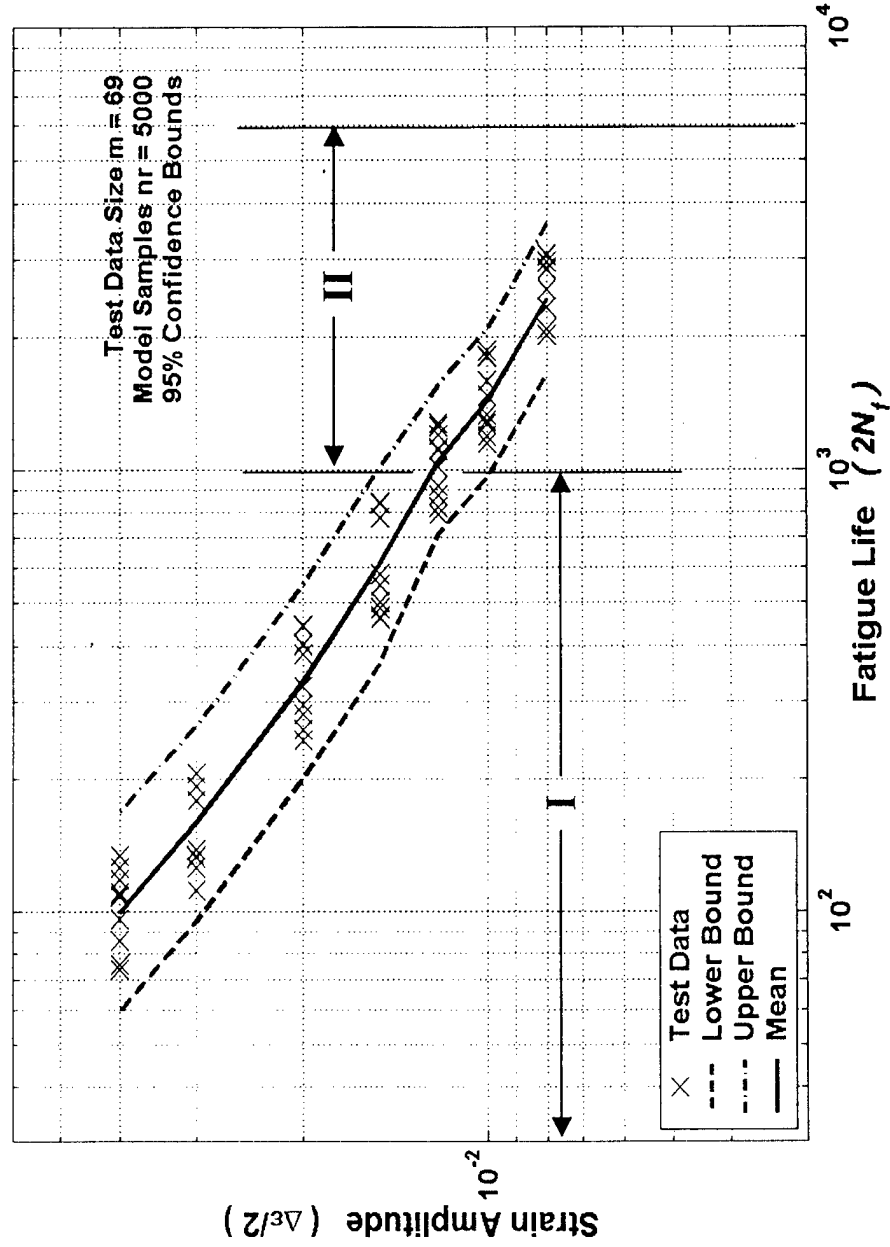
Strain-Life Data  
7050-T7451 Al, 2% Peak Load Drop



Reversals to Failure ( $2N_f$ )

# Prediction Intervals

## Probabilistic Strain-Life Model, LCF Region



## Conclusions & Further Research

- ASTM Hourglass Specimens Give Higher Mean, More Scatter than ASTM Uniform Gage Specimens
- Crack Growth in LCF Affects Scatter, Can Be Removed by Applying Peak Load Drop Failure Criteria
- Grain Orientation of ASTM Hourglass Specimens Affects Scatter, Causes Test Error
- Probabilistic Linear Regression Proves Acceptable for Strain-Life Characterization in LCF Region
- Incorporate More Robust Statistical Methods for HCF Portion of Strain-Life Curve
- Investigate Sensitivity of Component Fatigue Reliability Analyses to Parameters in Probabilistic Strain-Life Model